layer thickness may be written as

$$\delta_t = Rm^{-Q} \qquad Q > 0 \tag{7}$$

and vanishes in the limit $Rm = \infty$.

Although the formulations of Eqs. (5–7) appear to be arbitrary, as was the case of the inviscid boundary-layer thickness, velocity, and magnetic-field components of Ref. 1, nevertheless, they lead to consistent results.

The results from Ref. 1 which we need in our analysis are

$$\begin{aligned} U &= U_0 + U_1\,Rm^{-1/2} + \ldots \\ V &= V_1\,Rm^{-1/2} + \ldots \\ Hx &= Hx_0 + Hx_1\,Rm^{-1/2} + \ldots \\ Hy &= Hy_1\,Rm^{-1/2} + \ldots \end{aligned} \tag{8}$$

Introducing the set of equations [Eqs. (8)] as well as (5–7) into Eq. (4), we have

$$U_0\frac{\partial\theta_0}{\partial Y} + \ldots + V_1\frac{\partial\theta_0}{\partial Y}Rm^{(Q-1/2)} + \ldots = \frac{Pm}{Rm}\left[\frac{\partial Hy_1}{\partial X}Rm^{-1/2} + \ldots - \frac{\partial Hx_0}{\partial X}Rm^{+Q}\ldots\right]^2 \tag{9}$$

Thus it appears that, for $Q = \frac{1}{2}$, we are led to an equation, the coefficient of Rm^0 of boundary-layer character

$$U_0\frac{\partial\theta_0}{\partial X} + V_1\frac{\partial\theta_0}{\partial Y} = Pm\left[\frac{\partial Hx_0}{\partial Y}\right]^2 \tag{10}$$

Returning to the original dimensional variables, the magnetoinviscid-thermal boundary-layer equation is

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{1}{(4\pi)^2\sigma}\left(\frac{\partial h_x}{\partial y}\right)^2 \tag{11}$$

with the boundary condition $T = T_\infty$ for large y. Note that the temperature at the inner edge of the inviscid-thermal layer must be determined from the solution of Eq. (11).

Viscous-thermal sublayer

For the viscous-thermal region, we must examine all of the terms of Eq. (1). Before doing this, let us look at some of the important results from Ref. 1 for the viscous sublayer. First of all, it was shown that the viscous boundary layer with magnetic effects was of order $Re^{-1/2}$. Also, H_x is negligible, and H_y may be taken as constant across the viscous sublayer, its value having been calculated for the inner edge of the inviscid boundary layer. In addition,

$$j = -\tfrac{1}{4}(\partial h_x/\partial y) = \sigma u h_y \tag{12}$$

We see, then, that the boundary-layer analysis for the viscous-thermal region does not differ from the nonmagnetic case. The thermal boundary layer is of order $Re^{-1/2}Pr^{-1/2}$, and Eq. (1) becomes

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} + \rho\nu\left(\frac{\partial u}{\partial y}\right)^2 + \sigma u^2 h_y^2 \tag{13}$$

where u and v are the solutions of the viscous sublayer equations. The boundary condition is $T = T_w$ at the surface of the body and is prescribed; at the outer edge of the viscous-thermal layer, $T = T_i$, where T_i is found from the inner edge of the inviscid-thermal layer.

Equation (13) is nothing more than the thermal-energy equation that has been studied by Rossow,[4] Lykoudis,[5] and others in their study of boundary layers at low Rm with the applied magnetic field directed normal to the body surface. It is to be treated, though, after the inviscid-thermal boundary-layer equation [Eq. (11)] has been solved.

In conclusion, Eq. (11) is being studied now in terms of the similarity variable that was introduced by Sears in Ref. 1. What is needed, before a complete analysis can be made, is further study of the velocity and magnetic-field components as has been initiated by Sears and Mori.[6]

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Plane Poiseuille Flow of a Radiating and Conducting Gas

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Nomenclature

k = absorption coefficient
T^* = temperature
T_0^* = reference temperature, taken to be T_w^*
T = dimensionless temperature, T^*/T_0^*
y = distance from lower wall
q^* = total heat flux
q = dimensionless heat flux, $q^*/\sigma T_0^{*4}$
λ = thermal conductivity
τ^* = optical depth, $\int_0^y k\,dy$
τ = $3\tau^*/2$
σ = Stefan-Boltzmann constant
μ = viscosity
u = velocity
ϵ = $3\lambda k/4T_0^{*3}\tau_w^2$
δ = $\epsilon\tau_w^2$
ξ = y/y_w
ψ = $\mu u_M^2/y_w\sigma T_0^{*3}$

Subscripts

R = radiation
1 = lower wall
w = upper wall

In a previous paper,[1] the problem of Couette flow with a radiating and conducting gas was solved. It should also be pointed out that the rather broad class of radiation and conduction heat-transfer problems, which obey an energy equation of the form

$$(d/dy)[-\lambda(dT^*/dy) + q_R^*] = f(y) \tag{1}$$

that is, with variable dissipation, heat sources, etc., may also

Received March 15, 1965.

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be solved according to the techniques previously outlined. All symbols are defined in the nomenclature.

As a special case consider plane Poiseuille flow, so that the dissipation function $f(y)$ is given by[2]

$$f(y) = (\mu u M^2 16/yw^2)/[(2y/y_w) - 1]^2 \quad (2)$$

For this case, the problem reduces to the solution of the differential equation

$$(\epsilon d^2T/d\xi^2) - \epsilon\tau_w{}^2 T - T^4 =$$
$$-\alpha - \beta\tau_w\xi - (8\psi/\tau_w)(2\xi - 1)^2 + (\tau_w\psi/6)(2\xi - 1)^4 \quad (3)$$

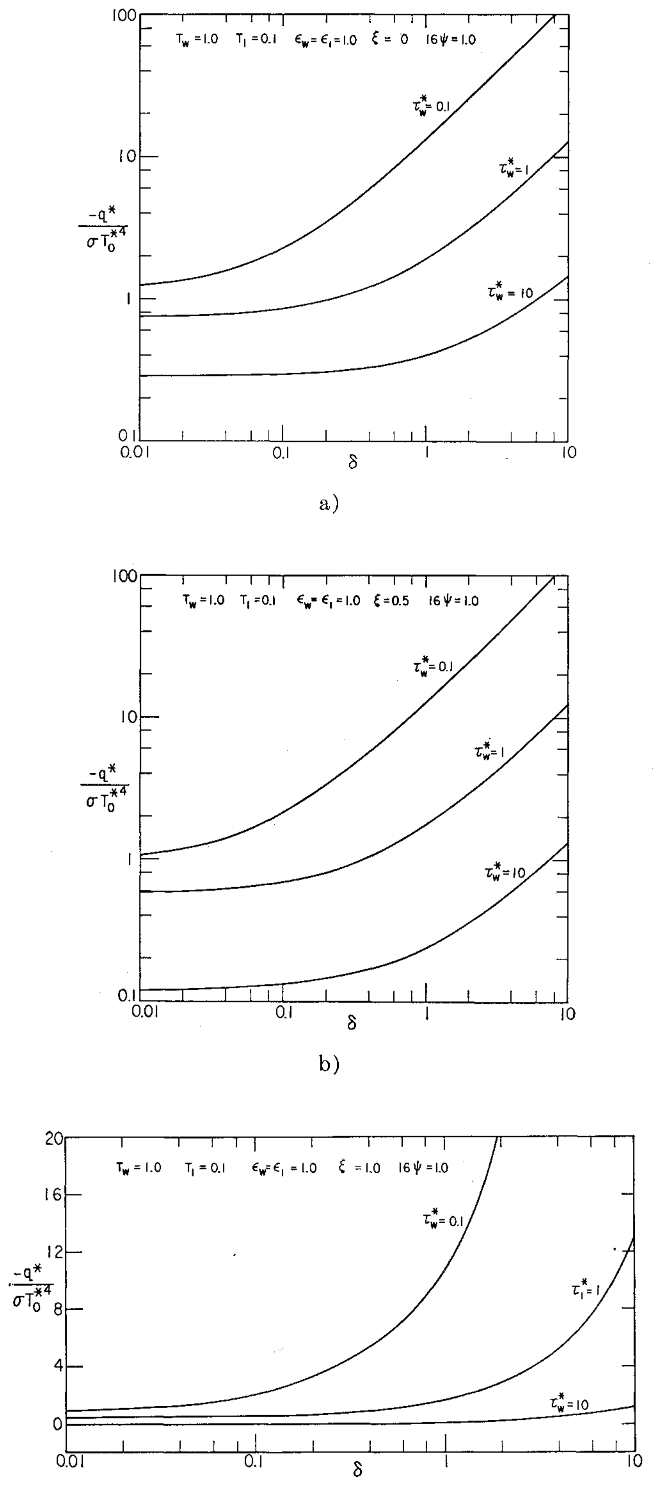

Fig. 1 Heat flux for radiating and conducting Poiseuille flow.

where

$$\alpha = \frac{1}{2 + \tau_w}\Big\{q^-(\tau_w) - q^+(0)[1 + \tau_w] + \delta\left[T_w + T_1(1 + \tau_w) + \frac{dT}{d\tau}\Big|_w - \frac{dT}{d\tau}\Big|_1 (1 + \tau_w)\right] - \frac{\tau_w\psi}{6}\left[-\tau_w - 10 - \frac{400}{\tau_w} - \frac{384}{\tau_w{}^2} + \frac{384}{\tau_w{}^3} + \frac{384}{\tau_w{}^4}\right]\Big\} \quad (4)$$

$$\beta = \frac{1}{2 + \tau_w}\Big\{q^-(\tau_w) - q^+(0) + \delta\left[T_w - T_1 + \frac{dT}{d\tau}\Big|_w + \frac{dT}{d\tau}\Big|_1\right] - \frac{64\psi}{\tau_w}\left[1 - \frac{1}{\tau_w{}^2}\right]\Big\} \quad (5)$$

$$\delta = \epsilon\tau_w{}^2 = \tfrac{3}{4}(\lambda k/\sigma T_0{}^{*3}) \quad (6)$$

$$\psi = \mu u M^2/y_w\sigma T_0{}^{*3} \quad (7)$$

The heat flux is given by

$$-q = 2\beta - 16\psi\,[-\tfrac{1}{6} + \xi - 2\xi^2 + \tfrac{4}{3}\xi^3] \quad (8)$$

Equation (3) may be solved for the following limiting cases: radiation dominant ($\epsilon \ll 1$), conduction dominant ($\epsilon \gg 1$), and for large optical depth ($\tau_w \gg 1$). The results for the heat flux are in good agreement with the radiation slip plus conduction approximation[3]

$$-q = \frac{T_w{}^4 - T_1{}^4}{1 + 3\tau_w{}^*/4} + \frac{(T_w - T_1)}{3\tau_w{}^*/4} - 16\psi\,[-\frac{1}{6} + \xi - 2\xi^2 + \frac{4}{3}\xi^3] \quad (9)$$

The heat flux for plane Poiseuille flow is plotted in Fig. 1 for unequal wall temperatures $T_w = 1$ and $T_1 = 0.1$, for $16\psi = 1$, and for values of ξ equal to 0, 0.5, and 1.

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Mass-Transfer Cooling of a Flat Plate with Various Transpiring Gases

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The purpose of this note is to provide mass-transfer cooling results for the flat plate, thereby supplementing information for stagnation flows previously published by the authors in Ref. 1. Consideration is given here to a laminar boundary-layer flow into which are injected various gases including hydrogen, helium, water vapor, argon, carbon dioxide, and xenon. The molecular weights of the aforementioned

Received February 23, 1965. The studies described herein were supported by a grant from the U.S. Air Force Office of Scientific Research, Mechanics Division.

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